

# Photon statistics measurement for coherent fields

Yuan Liu (刘源), Jun Wang (王俊), Shuangli Dong (董双丽),  
Guofeng Zhang (张国峰), Liantuan Xiao (肖连团), and Suotang Jia (贾锁堂)

State Key Laboratory of Quantum Optics and Quantum Optics Devices,  
College of Physics and Electronics Engineering, Shanxi University, Taiyuan 030006

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An efficient scheme for photon statistics measurement is presented based on the Hanbury-Brown-Twiss configuration. We set the sampling time  $T_s$  to satisfy the relationship of  $T_s < T_d < T_m$ , where  $T_d$  is the dead time of each detector and  $T_m$  is the laser pulse repetition period. And each single photon detector cannot detect more than one photon in each pulse. The approach can sufficiently eliminate the influences of the detector's dead time on photon statistics. At last, the photon statistics of coherent field is experimentally determined.

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Photon statistics measurement has been widely used to analyze quantum state of various light fields<sup>[1]</sup>. The complete description of the quantum state of a light field requires the knowledge of its density operator or Wigner function, but these quantities are hard to be measured with general experimental methods<sup>[2]</sup>. In order to study the nonclassical characteristic of light field such as the photon antibunching effect, Hanbury-Brown-Twiss (HBT)<sup>[3]</sup> configuration based on classical electromagnetic field theory is widely applied. By measuring the second-order correlation function  $g^{(2)}$ , the correlation of intensity light field fluctuation can be obtained<sup>[4-7]</sup>. The method<sup>[8,9]</sup> has been proved useful for studying characteristics of various light fields. However, the method cannot provide the information about the intensified fluctuations on timescale, and the photon statistics probabilities<sup>[10]</sup> are seriously affected by the statistics noise<sup>[7]</sup>.

In 2002, direct measurement of the photon statistics of a triggered single photon source was reported by Treussart *et al.*<sup>[11]</sup>. They used software for synchronous detection of the photons emitted from single molecules. The Mandel parameter  $Q$  was directly obtained from the photon statistic probability distribution which can be extracted by single event photon statistics measurement<sup>[12]</sup>. In this way, the real-time measurements for photon statistics avoid the effects of statistics noise. Nevertheless, after every photodetection event, the single photon detector needs some time (typically tens to hundreds nanoseconds) to recover for the next detection, which is defined as the dead time. It means that the detector cannot tell the difference between one and more photons within its dead time, which will result in serious error of the single event photon statistic probability measurement. In this paper, we present an exterior gating synchronizer counting scheme for photon statistics measurement. Compared with the software virtual sampling method, our approach overcomes the dead time's influence<sup>[13-15]</sup> on photon statistics.

Figure 1 shows the synchronizing sampling detection schematic diagram. The laser working as coherent field here is generated from a picosecond pulsed diode

laser (PicoQuant, PDL808) which is triggered by the transistor-transistor logic (TTL) pulse signals generated by a digital delay and pulse generator (SRS DG535). The laser pulse width is 50 ps at a central wavelength of 640 nm and the repetition rate is 1 MHz. In order to ensure the synchronism detection, the base clock signal from DG535 also provides a gate time for single photon detection. The incident photons are recollimated onto the active areas of two identical single photon counting modules (SPCM, Perkin Elmer SPCM-AQR-15) after partitioning by a 50/50 beam splitter (BS), which works as a standard HBT configuration. Then the output TTL pulses from the two SPCMs are fed into a homemade exterior gating logical circuit. The difference of the delay timers for the two SPCMs' output is used to compensate the time difference between two detection channels. When DG535 provides a time gate signal, the records within the time gates are considered as the detected signals while all records outside the time gates are rejected. Within a number of pulse cycles  $N_{\text{all}}$ , by recording each joint photodetection event from the two SPCMs, no photons counts  $N_0$ , only one photon counts  $N_1$ , and two photons counts  $N_2$  can be directly recorded by three independent counters. Then the probabilities of the number

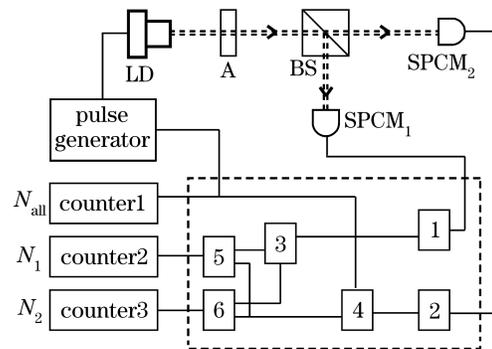


Fig. 1. Photon statistics measurement setup. LD: laser diode; A: attenuator; BS: non-polarizing beam splitter; SPCMs: single photon counting modules. The part in dashed line is our exterior gating circuit. 1, 2: nanosecond delay timer; 3, 4, 5: AND logical gate; 6: XOR logical gate.

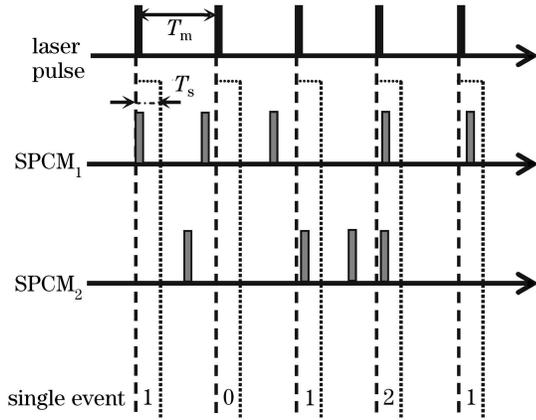


Fig. 2. Single event measurement with synchronization sampling.

of detected photons per pulse  $P(n)$  ( $n = 0, 1, 2$ ) can be calculated.

Figure 2 shows the schematic of single event photon statistics measurement. We set the gate duration time (sampling time)  $T_s$  to be shorter than the detector dead time  $T_d$  and much shorter than the laser pulse period  $T_m$ , namely,  $T_s < T_d < T_m$ . In this way, each SPCM gives no more than one count within each photodetection event. For a coherent light which contains  $\alpha$  detected photons per excitation pulse, the photon number probability distribution can be expressed as

$$P(n) = \frac{\alpha^n}{n!} e^{-\alpha}. \quad (1)$$

One can calculate the photon statistics probabilities of the Poissonian coherent state  $P_c(n)$ :

$$P_c(0) = P(0) = e^{-\alpha}, \quad (2)$$

$$P_c(1) = P(1) + 1/2P(2) + 1/4P(3) + \dots$$

$$= \sum_{k=1}^{\infty} \frac{1}{2^{k-1}} P(k) = 2e^{-\alpha/2}(1 - e^{-\alpha/2}), \quad (3)$$

$$P_c(2) = 1/2P(2) + 3/4P(3) + \dots$$

$$= \sum_{k=2}^{\infty} \frac{2^{k-1} - 1}{2^{k-1}} P(k) = (1 - e^{-\alpha/2})^2, \quad (4)$$

and

$$P_c(n, n > 2) = 0. \quad (5)$$

The mean photon number  $\bar{n}$  per detection pulse period is

$$\bar{n} = P_c(1) + 2P_c(2) = 2(1 - e^{-\alpha/2}). \quad (6)$$

From the experimental Mandel parameter  $Q$ :

$$Q \equiv \langle (\Delta n)^2 \rangle / \langle n \rangle - 1, \quad (7)$$

we obtain the theoretical Mandel parameter  $Q_c$ :

$$\begin{aligned} Q_c &= [P_c(0) \times \bar{n}^2 + P_c(1) \times (\bar{n} - 1)^2 \\ &\quad + P_c(2) \times (\bar{n} - 2)^2] / \bar{n} - 1 \\ &= e^{-\alpha/2} - 1 = -\bar{n}/2. \end{aligned} \quad (8)$$

In a typical experiment, for example, fine adjustment of the intensity of the laser (peak power is about 80 mW, frequency is 1 MHz) is possible by rotating a circular linear-wedge neutral density filter placed behind the diode laser, in order to attenuate the light to an average number of 0.1 photon per pulse at the detectors input. Correspondingly, the counting of each SPCM is about 50 kHz. Here, in 299613 periods (about 150 ms) there are 31356 recorded photons, which include 29732 one-photon events, 812 two-photon events and others are zero-photon events (empty). These data allow us to deduce the photon probabilities  $P(0) = 0.8981$ ,  $P(1) = 0.09923$ ,  $P(2) = 2.71 \times 10^{-3}$  and the mean number of detected photon per pulse  $\bar{n}_s = 0.1046$ , then obtain Mandel parameter  $Q = -0.05286$ .

From Eqs. (2)–(8), we can calculate the theoretical photon probabilities to a Poisson statistics coherent state  $C_{\text{theory1}}$ ,  $C_{\text{theory2}}$ , and  $C_{\text{theory3}}$  with the same average photon numbers. We inferred  $P(0) = 0.8981$ ,  $P(1) = 0.09912$ ,  $P(2) = 2.73 \times 10^{-3}$ , and the Mandel parameter  $Q_c = -\bar{n}_s/2 = -0.05230$ .

The comparison between the theoretical and experimental results is shown in Table 1. The experimental values are in good agreement with the theoretical predictions, which proves that the diode laser is an ideal Poissonian coherent source.

We found that the  $Q$  value is a little less than  $Q_c$ . The measurement error is about 0.5%. We also noted the same result in Ref. [11] where the measured value  $Q$  of the faint Ti:sapphire pulses was less than the predicted

**Table 1. Experimental Results of Photocount Probabilities  $P(n)$ , the Mean Number of Detected Photons per Pulse  $\bar{n}$ , and Mandel Parameter  $Q$  with Laser Peak Power  $P = 80, 120$ , and  $160$  mW, Respectively. Correspondingly, the Theoretical Prediction to the Poissonian Coherent State  $C_{\text{theory1}}$ ,  $C_{\text{theory2}}$ , and  $C_{\text{theory3}}$  from Eqs. (2)–(8)**

Type	$P(0)$	$P(1)$	$P(2)$	$\bar{n}$	$Q$
$P_1 = 80$ mW	0.8981	0.09923	$2.76 \times 10^{-3}$	0.1046	-0.05216
$C_{\text{theory1}}$	0.8981	0.09912	$2.73 \times 10^{-3}$	0.1046	-0.05230
$P_2 = 120$ mW	0.9016	0.09577	$2.59 \times 10^{-3}$	0.1009	-0.04963
$C_{\text{theory2}}$	0.9016	0.09581	$2.54 \times 10^{-3}$	0.1009	-0.05045
$P_3 = 160$ mW	0.9027	0.09211	$2.56 \times 10^{-3}$	0.09722	-0.04459
$C_{\text{theory3}}$	0.9051	0.09249	$2.36 \times 10^{-3}$	0.09722	-0.04861

$Q_c$  for Poisson statistics. Here, the influence of the detector's dark count can be negligible in this experiment. The main measurement error comes from the unbalance of two detection channels. In fact, for example, if the power difference  $\Delta\bar{n}$  between the two separate beams after the beam splitter is 8%, we can calculate the error to the measurement  $Q$  is  $\Delta Q = 1/2 \times (\Delta\bar{n})^2 / [\bar{n}(1) + \bar{n}(2)]^2 = 0.32\%$ <sup>[8]</sup>. Note that there is no influence on the results from the detection efficiency of SPCMs and other optical loss, because the reduction of the incidence photons does not change its statistics probability.

In conclusion, we have illustrated experimentally the photon statistics of picosecond pulsed diode laser. With no more than one photon detected per pulse simultaneously, the Mandel parameters are directly measured by use of the deduced probability distribution  $P(n)$  which inferred from the set of time tagged photon counts. We perform the photon statistical properties through single photon synchronized detection by using our low-cost homemade exterior gating circuit. The scheme enables us to reduce the error count from back light and influence of dead time, and allows us to perform sub-Poisson or super-Poisson statistics measurement for various pulse radiation sources.

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