Diffraction properties of four-petal Gaussian beams in uniaxially anisotropic crystal

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Propagation properties of polarized four-petal Gaussian beams along the optical axis of uniaxially anisotropic crystals were investigated. Based on the paraxially vectorial theory of beam propagation, analytic expressions of the diffraction light field were obtained. The effects of the anisotropy on the polarization properties of the diffracted four-petal Gaussian beams have also been explained by numerical method. The results elucidate that the linear polarization state and the symmetry of the incident beams cannot be kept during propagation in anisotropic crystals.

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The propagation of laser beams in anisotropic media has received much attention, due to the important role of anisotropic materials in optical devices and nonlinear optics. Many optical devices are based on the effects of anisotropic medium on the field, such as ploarizers, compensators and switches, which are currently employed in a large amount of experimental situations[1,2]. Principally, such a type of problem can be treated by solving Maxwell equations in anisotropic media[3], but sophisticated mathematical calculations must be performed. Recently, a general method for vectorially describing an optical beam paraxially propagation through a uniaxially anisotropic medium has been presented[4,5]. The basic idea of this method is that optical field inside the anisotropic media can be evaluated as a superposition of an ordinary and extraordinary beam. So the beam propagation equations in uniaxial anisotropic media can be resolved under the condition that the boundary field distribution is given on a reference plane. By using this method, some researchers have studied the propagation of Gaussian beams, Hermite-Cosh-Gaussian beams, Laguerre-Gaussian beams, Bessel-Gaussian beams, flattened Gaussian beams in uniaxial crystal, and so on[6−11]. In this letter, we will mainly study the propagation of a new type of laser array named four-petal Gaussian beams in uniaxially anisotropic crystals. Based on paraxial propagation equations in uniaxial crystals, the closed form propagation expressions for the four-petal Gaussian beams in uniaxial crystals are derived and we will give numerical calculation results and analysis.

We assume that the optical axis of an uniaxially anisotropic crystal coincides with the z-axis of a reference frame, and entrance surface of the crystal is the $z = 0$ plane. The dielectric tensor of the crystal can be written as

$$\varepsilon = \begin{pmatrix} n_\mathrm{o}^2 & 0 & 0 \\ 0 & n_\mathrm{o}^2 & 0 \\ 0 & 0 & n_\mathrm{e}^2 \end{pmatrix}, \tag{1}$$

where n_o and n_e are the ordinary and extraordinary refractive indexes, respectively. Subscripts o and e indicate ordinary and extraordinary quantities, respectively. The field of four-petal Gaussian beams at the incident plane $z = 0$ is[12]

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, 0) = E_0 \left(\frac{x_0 y_0}{w_0^2}\right)^{2n} \exp\left(-\frac{x^2+y^2}{w_0^2}\right)\vec{\mathbf{e}}_x, \quad n = 0, 1, 2, \cdots, \tag{2}$$

where n is the order of four-petal Gaussian beams and E_0 is a constant. $\vec{\mathbf{r}} = x\vec{\mathbf{e}}_x + y\vec{\mathbf{e}}_y$ is the position vector at any transverse planes. When $n = 0$, Eq. (2) reduces to a fundamental Gaussian beam with waist size w_0. In this work, we are interested only in the transverse electric field $\vec{\mathbf{E}} = E_x\vec{\mathbf{e}}_x + E_y\vec{\mathbf{e}}_y$ and it can be assumed that the boundary electric field at the plane $z = 0$ is polarized along the x axis. When the paraxial approximation is well satisfied, the longitudinal components can be neglected because it is much less than the transverse components. According to the discussion by Ciattoni *et al.*[4] the electric field propagating for a distance z through the crystal is given by

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, z) = \exp(ik_0 n_0 z)\left[\vec{\mathbf{A}}_\mathrm{o}(\vec{\mathbf{r}}, z) + \vec{\mathbf{A}}_\mathrm{e}(\vec{\mathbf{r}}, z)\right], \tag{3}$$

where $k_0 = 2\pi/\lambda$ is the wave number in vacuum. In Eq. (2), the vectorial slowly varying amplitudes $\vec{\mathbf{A}}_\mathrm{o}$ and $\vec{\mathbf{A}}_\mathrm{e}$ are defined as $\vec{\mathbf{A}}_\eta = A_{\eta x}\vec{\mathbf{e}}_x + A_{\eta y}\vec{\mathbf{e}}_y$ ($\eta = \mathrm{o}, \mathrm{e}$), and they can be expressed as follows:

$$\vec{\mathbf{A}}_\mathrm{o}(\vec{\mathbf{r}}, z) = \int \frac{\mathrm{d}^2\vec{\mathbf{k}}}{k^2}\begin{pmatrix} k_y^2 & -k_x k_y \\ -k_x k_y & k_x^2 \end{pmatrix} \times \breve{E}(\vec{\mathbf{k}})\exp\left(i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} - \frac{ik^2}{2k_0 n_\mathrm{o}}z\right), \tag{4}$$

$$\vec{\mathbf{A}}_\mathrm{e}(\vec{\mathbf{r}}, z) = \int \frac{\mathrm{d}^2\vec{\mathbf{k}}}{k^2}\begin{pmatrix} k_x^2 & k_x k_y \\ k_x k_y & k_y^2 \end{pmatrix} \times \breve{E}(\vec{\mathbf{k}})\exp\left(i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} - \frac{in_\mathrm{o}k^2}{2k_0 n_\mathrm{e}^2}z\right), \tag{5}$$

where $\vec{\mathbf{k}} = k_x\vec{\mathbf{e}}_x + k_y\vec{\mathbf{e}}_y$, and $\breve{E}(\vec{\mathbf{k}})$ is the two-dimensional (2D) Fourier transform of the transverse field at $z = 0$

$$\breve{E}(\vec{\mathbf{k}}) = \frac{1}{(2\pi)^2}\int \mathrm{d}^2\vec{\mathbf{r}}\exp(-i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},0). \quad (6)$$

Using the following equation[12]:

$$x^{2n} = \frac{(2n)!}{2^{3n}}\sum_{s=0}^{n}\frac{1}{(n-s)!\,(2s)!}H_{2s}\left(\sqrt{2}x\right), \quad (7)$$

with $H_j(\cdot)$ being the Hermite polynomial of order j, the substitution from Eq. (2) into Eq. (6) yields

$$\vec{\mathbf{E}}(\vec{\mathbf{k}}) = \frac{\left[(2n)!\right]^2 w_0^2}{4\pi 2^{6n}}\sum_{s=0}^{n}\sum_{t=0}^{n}\frac{(-1)^{s+t}}{(n-s)!\cdot(n-t)!\cdot(2s)!\cdot(2t)!}$$

$$\times H_{2s}\left(\frac{k_x w_0}{\sqrt{2}}\right)H_{2t}\left(\frac{k_y w_0}{\sqrt{2}}\right)\exp\left(-\frac{k^2 w_0^2}{4}\right)\hat{\mathbf{e}}_x. \quad (8)$$

Substituting Eq. (8) into Eqs. (4) and (5), and applying the equation

$$H_j(x) = \sum_{k=0}^{[j/2]}\frac{(-1)^k j!}{k!(j-2k)!}(2x)^{j-2k}. \quad (9)$$

Using the differentiation properties of the Fourier transform, we obtain

$$A_{\mathrm{o}}(r,z) = \frac{\left[(2n)!\right]^2 w_0^2}{2^{6n+1}}\sum_{s=0}^{n}\sum_{t=0}^{n}\sum_{p=0}^{s}\sum_{q=0}^{t}S(n,s,t,p,q)$$

$$\times\begin{pmatrix}\partial_x^{2s-2p}\partial_y^{2t-2q+1}\\ -\partial_x^{2s-2p+1}\partial_y^{2t-2q}\end{pmatrix}$$

$$\times\left\{\frac{y}{x^2+y^2}\left[1-\exp\left(-\frac{x^2+y^2}{Q_{\mathrm{o}}(z)}\right)\right]\right\}, \quad (10)$$

$$A_{\mathrm{e}}(r,z) = \frac{\left[(2n)!\right]^2 w_0^2}{2^{6n+1}}\sum_{s=0}^{n}\sum_{t=0}^{n}\sum_{p=0}^{s}\sum_{q=0}^{t}S(n,s,t,p,q)$$

$$\times\begin{pmatrix}\partial_x^{2s-2p+1}\partial_y^{2t-2q}\\ \partial_x^{2s-2p}\partial_y^{2t-2q+1}\end{pmatrix}$$

$$\times\left\{\frac{x}{x^2+y^2}\left[1-\exp\left(-\frac{x^2+y^2}{Q_{\mathrm{e}}(z)}\right)\right]\right\}, \quad (11)$$

where

$$S(n,s,t,p,q) = \frac{\left(\sqrt{2}w_0\right)^{2s+2t-2p-2q}}{(n-s)!\,(n-t)!\,(2s-2p)!\,(2t-2q)!p!q!}, \quad (12)$$

$Q_\eta = w_0^2 + \frac{2in_{\mathrm{o}}z}{k_0 n_\eta^2}$, $\eta = \mathrm{o,e}$; s, t, p, q are summation indices.

From the above equations, the field distributions of the four-petal beams at an arbitrary z-plane ($z > 0$) in the uniaxial crystal can be concluded if the boundary field $\vec{\mathbf{E}}(\vec{\mathbf{r}},0)$ is known. As a test, it seen that letting $n = 0$, the propagation equations of Gaussian beams can be obtained inside the crystal, which are consistent with the results in Ref. [6].

In this section, numerical calculations were performed using Eqs. (10)—(12) for the four-petal beams propagating in a rutile crystal. The calculation parameters are $n_{\mathrm{o}} = 2.616$, $n_{\mathrm{e}} = 2.903$, the wavelength $\lambda = 0.6328$ μm, and $w_0 = 20$ μm. Figure 1 gives the field distributions of four-petal Gaussian beams at the input plane $z = 0$ for $n = 0$, 2, 5 and 10, respectively. From Fig. 1 it can be seen that for nonzero values of n, the intensity distributions consist of four equal petals, which are spaced by $2n^{1/2}w_0$ in the x- and y-direction, respectively. In particular, the field distribution of four-petal Gaussian beam reduces to the fundamental Gaussian beam when the beam order $n = 0$ (see Fig. 1(a)).

During the propagation through the crystal, the four-petal Gaussian beams linearly polarized in the

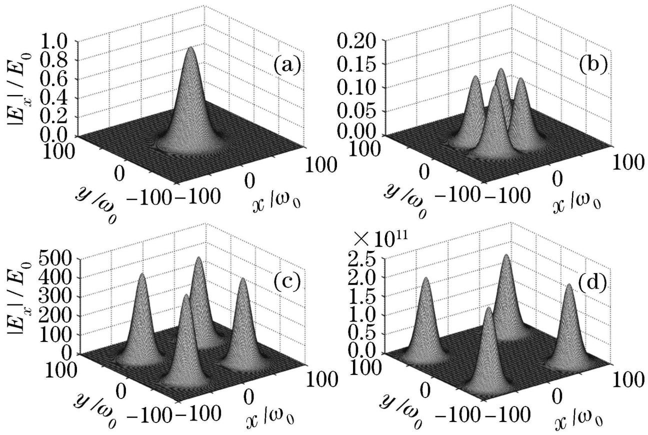

Fig. 1. Profiles of transverse field of the four-petal Gaussian beams linearly polarized in the x-direction with different orders in the input plane $z = 0$. (a) $n = 0$, (b) $n = 2$, (c) $n = 5$, (d) $n = 10$.

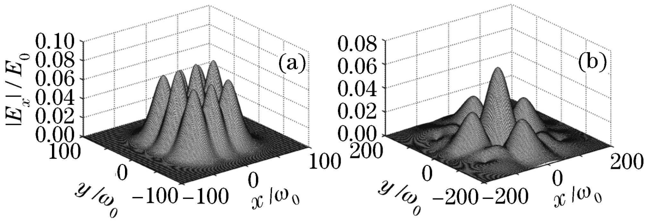

Fig. 2. Profiles of transverse field $|E_x|$ originated by an incident four-petal Gaussian beam of order $n = 2$ propagation in an uniaxial crystal at different planes (a) $z = Z_{\mathrm{R}}$, (b) $z = 4Z_{\mathrm{R}}$.

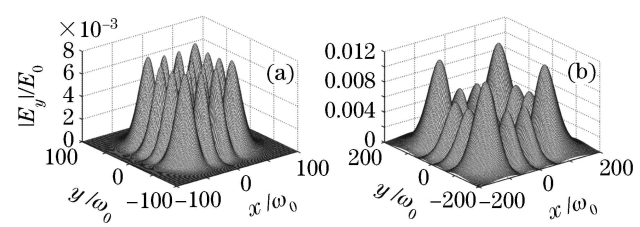

Fig. 3. Profiles of transverse field $|E_y|$ originated by an incident four-petal Gaussian beam of order $n = 2$ propagation in an uniaxial crystal at different planes (a) $z = Z_{\mathrm{R}}$, (b) $z = 4Z_{\mathrm{R}}$.

x-direction are split into ordinary and extraordinary parts. They propagate independently along the z-axis, and have x- and y-components. The transverse fields $|\mathbf{E}_x|$ and $|\mathbf{E}_y|$ at the plane (a) $z = Z_{\mathrm{R}}$, (b) $z = 4Z_{\mathrm{R}}$ are plotted in Figs. 2 and 3, respectively, where $Z_{\mathrm{R}} = \pi n_{\mathrm{o}} w_0^2/\lambda$ is the Rayleigh length of the ordinary beam. From Fig. 2 we find that, for the x-component $\mathbf{E}_x$, some other petals emerge among the four-petals, and the petals are of mirror symmetry with the central one on the z-axis. Figure 3 shows that y-component $\mathbf{E}_y$ of the electric field increases with the propagation distance z inside the crystal. Moreover, it vanishes along the x- and y-axes. So, the linear polarization property of the beam does not remain at any plane $z > 0$ in the rutile crystal, which is attributed to the anisotropic property of the crystal.

In conclusion, the propagation of four-petal Gaussian beams along the optical axis of uniaxially anisotropic crystal has been studied based on the paraxial vectorial theory. The analytical propagation equations have been derived, and the properties of four-petal Gaussian beams in uniaxial crystals have been illustrated with numerical examples. The ordinary and extraordinary beams originated by incident beams have different diffraction properties because of their different diffraction lengths in anisotropic crystal. As a result, the input beam loses both its boundary axial symmetry and its boundary linear polarization state.

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