Entanglement dynamics of W-like states in an open system

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The pairwise entanglement dynamics in a multipartite open system consisting of three entangled cavity photons locally coupled with independent N-mode reservoirs is studied via concurrence. The initial states of cavity photons are prepared in two types of W-like states while the corresponding reservoirs are prepared in the factorable vacuum state. The result shows that all the pairwise concurrences of the total system including cavities and reservoirs undergo qualitatively different dynamical behaviors. Among the two W-like states, only one could exhibit entanglement sudden death (ESD) leading the corresponding reservoirs to exhibit entanglement sudden birth. In addition, by taking the entanglement of the corresponding reservoirs into account, entanglement invariants are constructed for the W-like state that does not undergo ESD.

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Entanglement can be exploited to perform various intriguing global tasks in quantum computation and quantum communication because it possesses 'spooky' distance-independent nonlocality. However, in the process of entanglement distribution and qubit manipulation, each qubit is unavoidably exposed to its own uncontrollable environment, so the critical challenge for realizing quantum information processing (QIP) is the control of evolution of qubits in the presence of environmental noises and manipulation inaccuracies[1]. This control may be easily achieved for a single qubit, but for many qubits, entanglement dynamics has been a difficult subject and has attracted extensive interests in various contexts[2−12]. Recently, it has been shown that entanglement can be lost completely in a finite time despite of the fact that complete decoherence only occurs asymptotically. This phenomenon, named entanglement sudden death (ESD), has been theoretically predicted by Yu *et al.*[2], and experimentally observed for entangled photon pairs[13] and atomic ensembles[14]. Because an entangled state with ESD in evolution is less robust than states without it, ESD puts a limitation on the application time of entanglement. Therefore, studying ESD, especially conditions and parameter domains for its occurrence, is important from both theoretical and practical points of view.

The dynamics of the Greenberger-Horne-Zeilinger (GHZ) and W-like states which are regarded as two representatives of tripartite entangled states has been extensively studied recently[15]. Note particularly that W-like states, though being not maximally entangled, are proved to be strictly necessary in some tasks of QIP such as remote symmetric entangling[16], perfect teleportation of a qubit[17], etc. Hence, consideration of disentanglement dynamics of W-like states is of interest. It is well known that the total entanglement of a three-qubit system can generally be assessed by the pairwise entanglements and the so-called 3-tangle, a genuine three-way entanglement of the trio[18]. For W-like states, however, the 3-tangle is zero and remains as long as only local operations are performed. Therefore, it suffices to only explore the pairwise entanglements in studying entanglement dynamics of W-like states.

In Ref. [19], the authors structured an open quantum system which consisted of entangled cavity photons prepared in a Bell-like state $(\cos\theta|00\rangle_{\mathrm{c_1c_2}}+\sin\theta|11\rangle_{\mathrm{c_1c_2}})$ being affected by dissipation and its corresponding N-mode reservoirs. The result shows that the ESD of a bipartite cavity photons state is intimately linked to entanglement sudden birth (ESB) of the reservoirs. In other words, the presence of ESD implies the necessary apparition of ESB and the ESB can manifest before, simultaneously, or even after ESD. In this letter, we extend the model from two cavities to three cavities and choose two types of W-like states instead of Bell-like state as the initial state of cavities. By calculation and analyses, we find that among them only one could exhibit ESD leading the corresponding reservoirs to experience ESB. In addition, by taking the entanglement of the corresponding reservoirs into account, we find the entanglement invariants for the W-like state that does not undergo ESD in the open system.

We consider three identical cavities $\mathrm{c_1, c_2}$, and $\mathrm{c_3}$ interacting with three spatially separate N-mode reservoirs $\mathrm{r_1, r_2}$, and $\mathrm{r_3}$, respectively. Since each mode evolves independently, we can characterize the evolution of the overall system from the mode-reservoir dynamics. The interaction between a single cavity mode and an N-mode reservoir is described through the Hamiltonian ($\hbar$=1):

$$\hat{H}=\omega\hat{a}^{+}\hat{a}+\sum_{k=1}^{N}\omega_k\hat{b}^{+}\hat{b}+\sum_{k=1}^{N}g_k(\hat{a}\hat{b}_k^{+}+\hat{b}_k\hat{a}^{+}). \quad (1)$$

Let us consider the case when a cavity mode contains a single photon and its corresponding reservoir is in the vacuum state,

$$|\phi_0\rangle=|1\rangle_{\mathrm{c}}\otimes|\bar{0}\rangle_{\mathrm{r}}, \quad (2)$$

with the state $|\bar{0}\rangle_{\mathrm{r}}=\prod_{k=1}^{N}|0_k\rangle_{\mathrm{r}}$. Then the evolution given by Eq. (1) leads to the state

$$|\phi_t\rangle_{\mathrm{cr}}=\xi(t)|1\rangle_{\mathrm{c}}|\bar{0}\rangle_{\mathrm{r}}+\sum_{k=1}^{N}\lambda_k(t)|0\rangle_{\mathrm{c}}|1_k\rangle_{\mathrm{r}}, \quad (3)$$

where the state $|1_k\rangle_r$ accounts for the reservoir having one photon in mode k. The amplitude $\xi(t)$ converges to $\xi(t) = \exp(-kt/2)$ in the limit of $N \to \infty$ for a reservoir with a flat spectrum. If we define the normalized collective state with one excitation in the reservoir as

$$|\bar{1}\rangle_{\mathrm{r}} = \frac{1}{\chi(t)} \sum_{k=1}^{N} \lambda_k(t) |1_k\rangle_{\mathrm{r}}, \tag{4}$$

Eq. (3) can be rewritten as

$$|\phi_t\rangle_{\mathrm{cr}} = \xi(t)|1\rangle_{\mathrm{c}}|\bar{0}\rangle_{\mathrm{r}} + \chi(t)|0\rangle_{\mathrm{c}}|\bar{1}\rangle_{\mathrm{r}}, \tag{5}$$

here the amplitude $\chi(t)$ converges to the expression $\chi(t) = (1 - \exp(-kt))^{1/2}$ in the large N limit. Described in this way the cavity and reservoir evolve as an effective two-qubit system.

In two-qubit domains, there exist a number of good measures of entanglement such as concurrence[20] and negativity[21]. Although the various entanglement measures may be somewhat different quantitatively, they are qualitatively equivalent to each other in the sense that all of them are equal to zero for separate states. Here we adopt the concurrence as the measure of entanglement which applies to both pure and mixed states. The concurrence C for the reduced density matrix ρ of a two-qubit system is defined as

$$C(\rho) = \max\{0, \lambda_1^{1/2} - \lambda_2^{1/2} - \lambda_3^{1/2} - \lambda_4^{1/2}\}, \tag{6}$$

where λ_i (i=1–4) with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of the matrix $\varsigma = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. Here σ_y is a Pauli matrix and ρ^* is the complex conjugation of ρ. For a system of two qubits, there are so-called X-form states whose density matrix ρ is of the form

$$\rho = \begin{pmatrix} x & 0 & 0 & v \\ 0 & y & u & 0 \\ 0 & u^* & z & 0 \\ v^* & 0 & 0 & w \end{pmatrix}, \tag{7}$$

with x, y, z, w are real positive and u, v are complex quantities. The X-class states have the property that the corresponding two-qubit density matrix preserves the X-form during the system evolution. For the X-state equation (7), the concurrence can be derived as

$$C(\rho) = 2\max\{0, |u| - \sqrt{xw}, |v| - \sqrt{yz}\}. \tag{8}$$

The two types of normalized W-like states as the initial state of entangled cavity photons are of the form

$$\begin{aligned} |\Phi\rangle_{\mathrm{c_1c_2c_3}} &= (\cos(\theta)|001\rangle \\ &+ \sin(\theta)\sin(\varphi)|010\rangle + \sin(\theta)\cos(\varphi)|100\rangle)_{\mathrm{c_1c_2c_3}}, \end{aligned} \tag{9}$$

and

$$\begin{aligned} |\Psi\rangle_{\mathrm{c_1c_2c_3}} &= (\cos(\theta)|110\rangle \\ &+ \sin(\theta)\sin(\varphi)|101\rangle + \sin(\theta)\cos(\varphi)|011\rangle)_{\mathrm{c_1c_2c_3}}, \end{aligned} \tag{10}$$

with θ and φ being real parameters. The pairwise entanglements of these two types of W-like states for all bipartite subsystems, in terms of concurrence, are the same initially. However, as we shall show below, they evolve in time in totally different ways.

We choose the state of Eq. (9) as the initial state of cavities subsystem $\mathrm{c_1c_2c_3}$ and assume all their corresponding reservoirs to be initially in the vacuum. Hence, the total state of the system at $t = 0$ is

$$|\Phi_0\rangle = |\Phi\rangle_{\mathrm{c_1c_2c_3}} \otimes |\overline{000}\rangle_{\mathrm{r_1r_2r_3}}, \tag{11}$$

where $|\overline{000}\rangle_{\mathrm{r_1r_2r_3}} \equiv |\bar{0}\rangle_{\mathrm{r_1}} \otimes |\bar{0}\rangle_{\mathrm{r_2}} \otimes |\bar{0}\rangle_{\mathrm{r_3}}$. According to Eq. (5), the evolution of the overall system will be given by

$$\begin{aligned} |\Phi_t\rangle &= \cos(\theta)|00\rangle_{\mathrm{c_1c_2}}|\overline{00}\rangle_{\mathrm{r_1r_2}}|\phi_t\rangle_{\mathrm{c_3r_3}} \\ &+ \sin(\theta)\sin(\varphi)|00\rangle_{\mathrm{c_1c_3}}|\phi_t\rangle_{\mathrm{c_2r_2}}|\overline{00}\rangle_{\mathrm{r_1r_3}} \\ &+ \sin(\theta)\cos(\varphi)|\phi_t\rangle_{\mathrm{c_1r_1}}|00\rangle_{\mathrm{c_2c_3}}|\overline{00}\rangle_{\mathrm{r_2r_3}}. \end{aligned} \tag{12}$$

The reduced density matrix $\rho^{\Phi}_{\mathrm{c_1c_2}}$ can be obtained by tracing over the degrees of freedom of qubits $\mathrm{c_3, r_1, r_2}$, and $\mathrm{r_3}$, which remains the X-form with

$$\begin{aligned} x &= \cos^2(\theta)\xi^2(t) + \chi^2(t), \\ y &= \sin^2(\theta)\sin^2(\varphi)\xi^2(t), \\ z &= \sin^2(\theta)\cos^2(\varphi)\xi^2(t), \\ \omega &= v = 0, \\ u &= \frac{1}{2}\sin^2(\theta)\sin(2\varphi)\xi^2(t). \end{aligned} \tag{13}$$

By virtue of Eqs. (8) and (13), the corresponding concurrence is

$$C(\rho^{\Phi}_{\mathrm{c_1c_2}}(t)) = \sin^2(\theta)\sin(2\varphi)\mathrm{e}^{-kt}. \tag{14}$$

In addition to considering the entanglement evolution of the cavities subsystem, it is also of interest to investigate the entanglement of the reservoirs involved. As a complement to the cavity pair $\mathrm{c_1c_2}$, we derive the concurrence $C(\rho^{\Phi}_{\mathrm{r_1r_2}})$ for the reduced density matrix $\rho^{\Phi}_{\mathrm{r_1r_2}}$ whose form is the same as $\rho^{\Phi}_{\mathrm{c_1c_2}}$ but with the following matrix elements:

$$\begin{aligned} x &= \cos^2(\theta)\chi^2(t) + \xi^2(t), \\ y &= \sin^2(\theta)\sin^2(\varphi)\chi^2(t), \\ z &= \sin^2(\theta)\cos^2(\varphi)\chi^2(t), \\ \omega &= v = 0, \\ u &= \frac{1}{2}\sin^2(\theta)\sin(2\varphi)\chi^2(t). \end{aligned} \tag{15}$$

The concurrence of $\rho^{\Phi}_{\mathrm{r_1r_2}}$ is thus

$$C(\rho^{\Phi}_{\mathrm{r_1r_2}}(t)) = \sin^2(\theta)\sin(2\varphi)(1 - \mathrm{e}^{-kt}). \tag{16}$$

In Fig. 1 we plot the time-evolution of $C(\rho^{\Phi}_{\mathrm{c_1c_2}})$ and $C(\rho^{\Phi}_{\mathrm{r_1r_2}})$ for several parameters of the cavity's initial state. We see that the entanglement $C(\rho^{\Phi}_{\mathrm{c_1c_2}})$ decreases asymptotically and vanishes only at $t \to \infty$ while the entanglement $C(\rho^{\Phi}_{\mathrm{r_1r_2}})$ increases asymptotically and begins to appear from $t = 0$. This implies nonexistence of ESD of the cavities and ESB of two reservoirs. Moreover, the loss of entanglement $C(\rho^{\Phi}_{\mathrm{c_1c_2}})$ for the system $\mathrm{c_1c_2}$ is instantly compensated by concurrence gain in $C(\rho^{\Phi}_{\mathrm{r_1r_2}})$ for the reservoirs, that is the entanglement transfer from the subsystem $\mathrm{c_1c_2}$ to the subsystem $\mathrm{r_1r_2}$ in such a way that

$$\begin{aligned} C(\rho^{\Phi}_{\mathrm{c_1c_2}}(t)) + C(\rho^{\Phi}_{\mathrm{r_1r_2}}(t)) &= \sin^2(\theta)\sin(2\varphi) \\ &= C(\rho^{\Phi}_{\mathrm{c_1c_2}}(0)). \end{aligned} \tag{17}$$

Owing to the symmetry of the model, the time-evolution of $C^{\Phi}_{c_1c_3}, C^{\Phi}_{c_2c_3}, C^{\Phi}_{r_1r_3}$, and $C^{\Phi}_{r_2r_3}$ are analogous as above studied and satisfy the following invariants:

$$C(\rho^{\Phi}_{c_1c_3}(t)) + C(\rho^{\Phi}_{r_1r_3}(t)) = \cos(\varphi)\sin(2\theta) = C(\rho^{\Phi}_{c_1c_3}(0)), \tag{18}$$

$$C(\rho^{\Phi}_{c_2c_3}(t)) + C(\rho^{\Phi}_{r_2r_3}(t)) = \sin(\varphi)\sin(2\theta) = C(\rho^{\Phi}_{c_2c_3}(0)). \tag{19}$$

The entanglement invariants presented in Eqs. (17)–(19) suggest some possible kinds of 'entanglement conservation', which can be interpreted as a process of entanglement transfer from cavities to the corresponding reservoirs. Entanglement conservation is an open issue that is largely unexplored, and one cannot expect conservation of entanglement in the sense of dynamical conservation laws, since entanglement is not defined as an observable or represented by a Hermitian operator[7]. The entanglement decay (creation) of cavities (reservoirs) is in essence due to the local interactions between cavities and the corresponding reservoirs. This interaction leads to the local decoherece of the cavity, which in turn destroys the global entanglement of the cavities. On the other hand, these local interactions can also produce the entanglement between the cavity and the local reservoir,

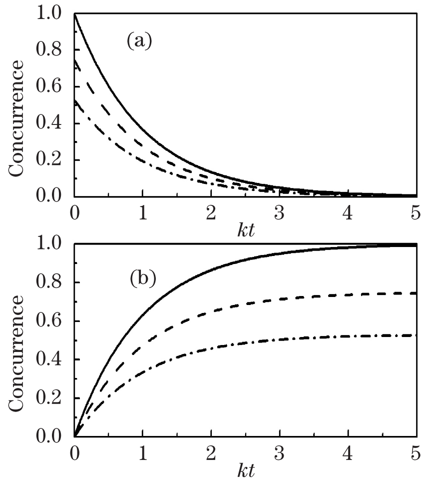

Fig. 1. (a) Concurrence of the cavities c_1, c_2 and (b) concurrence of the reservoirs r_1, r_2, both for the choice of Eq. (9), as a function of the dimensionless time kt for various parameters: $\theta = \pi/2, \varphi = \pi/4$ (solid line); $\theta = \pi/3, \varphi = \pi/4$ (dashed line); $\theta = \pi/3, \varphi = \pi/8$ (dashed dot line).

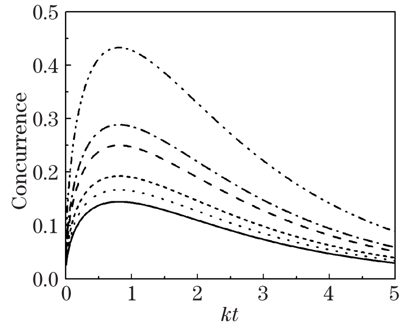

Fig. 2. Evolution of other pairwises entanglement: $C_{c_1r_1}$ (solid line); $C_{c_1r_2}$ (dashed line); $C_{c_1r_3}$ (dotted line); $C_{c_2r_3}$ (dashed dot line); $C_{c_2r_2}$ (dot-dot-dashed line); $C_{c_3r_3}$ (short-dashed line) for the initial state of Eq. (9) with $\theta = \varphi = \pi/3$.

and are responsible for the newly created entanglement of remote cavity and reservoirs. In Fig. 2, we plot the pairwise entanglement evolution between the cavities and the reservoirs in both the local and remote positions.

Considering the interaction between the cavities and its reservoirs, the initial cavity's entanglement rests in the spreading out of entanglement over all of the system's degrees of freedom that may become entangled with each other in all possible ways. So, by calculating in the same way, we can acquire the rest of pairwises entanglement, namely: $C^{\Phi}_{c_1r_1}, C^{\Phi}_{c_1r_2}, C^{\Phi}_{c_1r_3}, C^{\Phi}_{c_2r_2}, C^{\Phi}_{c_3r_3}$, and $C^{\Phi}_{c_2r_3}$. Considering that the factor $\xi(t)\chi(t)$ is included in their expressions, all the evolutions of these pairwises entanglement are similar (see Fig. 2).

As a comparison, in the following, we choose Eq. (10) to be the initial cavities state. Then the total system density matrix at $t = 0$ is

$$|\Psi_0\rangle = |\Psi\rangle_{c_1c_2c_3} \otimes |\overline{000}\rangle_{r_1r_2r_3}. \tag{20}$$

The evolved state of the total system at time t can be expressed in the same standard basis as

$$\begin{aligned}|\Psi_t\rangle &= \cos(\theta)|\phi_t\rangle_{c_1r_1}|\phi_t\rangle_{c_2r_2}|0\rangle_{c_3}|\bar{0}\rangle_{r_3} \\ &+ \sin(\theta)\sin(\varphi)|\phi_t\rangle_{c_1r_1}|0\rangle_{c_2}|\bar{0}\rangle_{r_2}|\phi_t\rangle_{c_3r_3} \\ &+ \sin(\theta)\cos(\varphi)|0\rangle_{c_1}|\bar{0}\rangle_{r_1}|\phi_t\rangle_{c_2r_2}|\phi_t\rangle_{c_3r_3}.\end{aligned} \tag{21}$$

In this case, the reduced density matrices $\rho^{\Psi}_{c_1c_2}$ and $\rho^{\Psi}_{r_1r_2}$ remain in the X-form with

$$\begin{gathered}x = \cos^2(\theta)\chi^4(t) + \sin^2(\theta)\chi^2(t), \\ y = \cos^2(\theta)\chi^2(t)\xi^2(t) + \sin^2(\theta)\cos^2(\varphi)\xi^2(t), \\ z = \cos^2(\theta)\chi^2(t)\xi^2(t) + \sin^2(\theta)\sin^2(\varphi)\xi^2(t), \\ w = \cos^2(\theta)\xi^4(t), v = 0, \\ u = \sin^2(\theta)\sin(\varphi)\cos(\varphi)\xi^2(t),\end{gathered} \tag{22}$$

and

$$\begin{gathered}x = \cos^2(\theta)\xi^4(t) + \sin^2(\theta)\xi^2(t), \\ y = \cos^2(\theta)\chi^2(t)\xi^2(t) + \sin^2(\theta)\cos^2(\varphi)\chi^2(t), \\ z = \cos^2(\theta)\chi^2(t)\xi^2(t) + \sin^2(\theta)\sin^2(\varphi)\chi^2(t), \\ w = \cos^2(\theta)\chi^4(t), v = 0, \\ u = \sin^2(\theta)\sin(\varphi)\cos(\varphi)\chi^2(t).\end{gathered} \tag{23}$$

From Eqs. (22), (23), and (8), we obtain

$$\begin{aligned}C(\rho^{\Psi}_{c_1c_2}) &= \max\{0, \sin^2(\theta)|\sin(2\varphi)|\xi^2(t) \\ &- 2|\cos(\theta)|\xi^2(t)\chi(t)\sqrt{\cos^2(\theta)\chi^2(t) + \sin^2(\theta)}\},\end{aligned} \tag{24}$$

and

$$\begin{aligned}C(\rho^{\Psi}_{r_1r_2}) &= \max\{0, \sin^2(\theta)|\sin(2\varphi)|\chi^2(t) \\ &- 2|\cos(\theta)|\chi^2(t)\xi(t)\sqrt{\cos^2(\theta)\xi^2(t) + \sin^2(\theta)}\}.\end{aligned} \tag{25}$$

To visualize the time evolution, we plot $C(\rho^{\Psi}_{c_1c_2})$ and $C(\rho^{\Psi}_{r_1r_2})$ as functions of rescaled time kt for some definite initial states of cavities c_1, c_2, c_3 with different parameters in Fig. 3. From Fig. 3(a) we see that the entanglement between cavities c_1 and c_2 can abruptly vanishes in a finite time and the length of the time interval for the zero entanglement is dependent on the degree of entanglement of the initial state, namely, the smaller the amount of initial cavity's entanglement, the shorter the time at which $C(\rho^{\Psi}_{c_1c_2})$ reaches zero. We also see that the sudden birth of entanglement arises between the two reservoirs when the entanglement between the two cavities suddenly disappears. Note that the entanglement contained initially in the cavity-cavity subsystem is transferred to the corresponding reservoir subsystem but the sum $C(\rho^{\Psi}_{c_1c_2})+C(\rho^{\Psi}_{r_1r_2})$ is not preserved as for the choice of Eq. (9) considered above. The time for which ESD and ESB occur can be calculated from Eqs. (24) and (25) as

$$t_{\mathrm{ESD}} = -\frac{1}{k}\ln[1-\frac{1}{2}\tan^2(\theta)(\sqrt{1+\sin^2(2\varphi)}-1)],$$
$$t_{\mathrm{ESB}} = -\frac{1}{k}\ln[\frac{1}{2}\tan^2(\theta)(\sqrt{1+\sin^2(2\varphi)}-1)]. \quad (26)$$

Obviously, from Eq. (26) we learn that ESB occurs for $\tan^2(\theta)(\sqrt{1+\sin^2(2\varphi)}-1) < 2$, as is the case for ESD. Furthermore, when $\csc^2(\theta) = \sqrt{1+\sin^2(2\varphi)}$, $t_{\mathrm{ESD}} = t_{\mathrm{ESB}}$, that is, ESD and ESB happen simultaneously; when $\csc^2(\theta) > (<)\sqrt{1+\sin^2(2\varphi)}$, ESB can occurs after (before) ESD. For example, if $\varphi = \pi/4$, then $\theta \approx 57°$, we obtain $t_{\mathrm{ESD}} = t_{\mathrm{ESB}} = k^{-1}\ \ln(2)$, as shown in Fig. 4.

Making use of the same way above, we can calculate other concurrences of cavity-cavity (or reservoir-reservoir) subsystem, namely: $c_1\otimes c_3, c_2\otimes c_3$ ($r_1\otimes r_3, r_2\otimes r_3$) and find that the evolution of these concurrences is very resemble with that of $c_1\otimes c_2$ and $r_1\otimes r_2$. Simultaneously, to study how the entanglement is shared among the parties in this case, we also acquire other concurrences of two qubits between cavities and reservoirs, namely $C^{\Psi}_{c_1r_1}, C^{\Psi}_{c_1r_2}, C^{\Psi}_{c_1r_3}, C^{\Psi}_{c_2r_2}, C^{\Psi}_{c_3r_3}, C^{\Psi}_{c_2r_3}$, however, in contrast with the first case (Fig. 2), the dynamics evolution of these concurrences is very slight. In particular for $C^{\Psi}_{c_1r_2}, C^{\Psi}_{c_1r_3}, C^{\Psi}_{c_2r_3}$, the degree of entanglement is

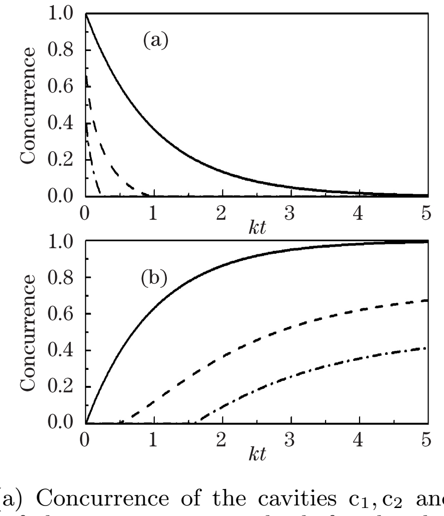

Fig. 3. (a) Concurrence of the cavities c_1, c_2 and (b) concurrence of the reservoirs r_1, r_2, both for the choice of Eq. (10), as a function of the dimensionless time kt for various parameters: $\theta = \pi/2, \varphi = \pi/4$ (solid line); $\theta = \pi/3, \varphi = \pi/4$ (dashed line); $\theta = \pi/4, \varphi = \pi/4$ (dashed dot line).

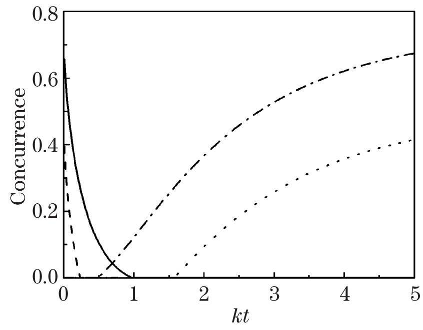

Fig. 4. Evolution of $C_{c_1c_2}$ and $C_{r_1r_2}$ for the initial state of Eq. (10): $C_{c_1c_2}$ (solid line), $C_{r_1r_2}$ (dashed dot line) with $\theta = \pi/3, \varphi = \pi/4$; $C_{c_1c_2}$ (dashed line), $C_{r_1r_2}$ (dotted line) with $\theta = \pi/4, \varphi = \pi/4$.

remained zero until both θ and φ are very small, that is, the entanglement contained initially between the cavities ($c_1\otimes c_2, c_1\otimes c_3, c_2\otimes c_3$) flows a little into the joint-system of $c_1\otimes r_2, c_1\otimes r_3, c_2\otimes r_3$ along the evolution of the total system.

In conclusion, we have investigated the dynamics of disentanglement of tripartite W-like states in an open system where three entangled cavity photons are coupled locally with three independent N-mode reservoirs. We have considered two initial conditions determined by two types of W-like states given by Eqs. (9) and (10) but the same vacuum state of the reservoirs. Among the two cases, the loss of pairwise entanglement is related to the birth of entanglement between the corresponding reservoirs and only one W-like state could exhibit ESD leading the corresponding reservoirs to experience ESB. Physically, this is due to the assumption that the reservoirs are initially in vacuum. Therefore, unexcited cavities do not interact with the reservoirs, while only excited ones do. Obviously, the larger the number of the initial excited cavities, the stronger is the cavity-reservoir coupling and, thus, the quicker is the transfer of entanglement from the cavities to the reservoirs, i.e., the sooner the vanishing of the entanglement. Since at $t = 0$ the number of excited cavities is one for state (9) and two for state (10), the latter interacts with the cavities stronger than the former. That is why ESD may occur for the latter but does not happen for the former. In addition, by taking the entanglement of the corresponding reservoirs into account, the entanglement invariants for the W-like state that does not undergo ESD are constructed. As a matter of practical application, our results suggest that it is preferable to use the state of Eq. (9) rather than those of Eq. (10) in QIP. Since the W-like states have a lot of unique applications, studying the W-like states including their entanglement dynamics is necessary.

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