General complementary media: electromagnetically transforming a small rectangle object to a large convex/concave pentagon object

Zhihong Chen (陈志红)*, Lihui Wang (王丽慧), Chunmei Wang (汪春梅), and Zuhua Fang (方祖华)

College of Information, Mechanical and Electrical Engineering, Shanghai Normal University, Shanghai 201418, China

**Corresponding author: chenzhihong98@163.com*

Received August 5, 2010; accepted October 14, 2010; posted online January 28, 2011

Based on the complementary media theory, we propose a way to transform a small object into a large one by coating it with a metamaterial shell with negative refractive index. Small waveguides made with this structure can replace large virtual tunnels to connect two separated waveguides. A small portal coating with complementary media is also designed in which a large entrance is concealed from electromagnetic wave detection. Two-dimensional numerical simulations are performed to illustrate the designed devices.

OCIS codes: 160.1190, 160.3918, 230.4320, 260.2710.

doi: 10.3788/COL201109.021601.

In the past few years, studies on cloaks have attracted much attention[1−33], since Pendry *et al.*[2] and Leonhardt[3] first described the concept of transformation optics in which coordinate transformations of electromagnetic (EM) waves can be physically realized through a complex medium. Based on the form-invariant property of Maxwell's equations, the coordinate transformation squeezes space into a shell so that the object inside the concealment volume is concealed. Cummer *et al.* first reported full-wave EM simulations of this cloaking structure in cylindrical version using ideal and non-ideal (but physically realizable) EM parameters[4]. With low-index EM metamaterials, a microwave invisible cloak was soon experimentally developed[5]. Apart from a cloak of invisibility, several other EM devices with novel functions, such as field-rotating device, wave concentrator, beam splitter, twisted EM wave, beam expander, and antenna, can also be produced with transformation optics[30−32].

Recently, another interesting device, a superscatterer, was proposed by Yang *et al.* based on the concept of complementary media[34]. It presents an object that looks like a scatterer bigger than its geometric size in EM wave detection. Ng *et al.* designed a frequency-selective superabsorber metamaterial, which could be constructed using an absorbing core material coated with a shell of isotropic double-negative metamaterial[35]. Lai *et al.* proposed another novel invisible cloak operating at a finite frequency that can cloak an object with a prespecified shape and size within a certain distance outside the shell[36,37]. Based on the mechanism of complementary media, Zhang *et al.* designed an "invisible tunnel" between two waveguide ports to make the EM wave propagate from one port to another in the open space through this tunnel[38].

In this letter, the coordinate transformation approach is used to transform a small object into a larger one based on complementary media. A rectangular coating with negative refractive index is designed using a metamaterial shell to produce a large convex (or concave) pentagon waveguide. Such a device can also conceal a large convex (or concave) pentagon entrance. The properties of our designs are demonstrated using the finite-element method.

In Fig. 1, a small rectangular object (gray rectangle) is covered by complementary media consisting of four parts marked with different color. The small rectangle can be amplified to a large convex (or concave) pentagon (dash-dot line region). The parameters are as follows: $|OA|=a_1$, $|OB|=a_2$, $|OC|=a_3$, $|CD|=L$, $|OE|=b_1$, $|OF|=b_2$, and $|OG|=b_3$. The parameters of the complementary media can be obtained using the transformation optics theory. The vacuum region (the convex or concave pentagon outside the rectangular object and the complementary media) is folded to the region of the complementary region. Simultaneously, the small rectangular object is expanded into a convex (or concave) pentagon. Therefore, the transformation equations for Region I are as follows:

$$\begin{cases} x' = -\dfrac{a_2-a_1}{a_3-a_2}\Big[x \pm \dfrac{L}{b_3}(b_3-y)\Big] + a_2\dfrac{a_3-a_1}{a_3-a_2} \\ y' = -\dfrac{a_2-a_1}{a_3-a_2}y + a_2\dfrac{a_3-a_1}{a_3-a_2}\dfrac{y}{x \pm \frac{L}{b_3}(b_3-y)} \\ z' = z \end{cases}. \quad (1)$$

Using coordinate transformation, the relative permittivity $\bar{\varepsilon}$ and permeability $\bar{\mu}$ in the transformation region are achieved as follows:

$$\frac{\bar{\bar{\varepsilon}}}{\varepsilon} = \frac{\bar{\bar{\mu}}}{\mu} = \begin{bmatrix} \dfrac{a^2+b^2}{ad-bc} & \dfrac{ac+bd}{ad-bc} & 0 \\ \dfrac{ac+bd}{ad-bc} & \dfrac{c^2+d^2}{ad-bc} & 0 \\ 0 & 0 & \dfrac{1}{ad-bc} \end{bmatrix}, \quad (2)$$

where $a = \dfrac{\partial x'}{\partial x} = -\dfrac{a_2-a_1}{a_3-a_2}$,

$$b = \frac{\partial x'}{\partial y} = \pm\frac{L}{b_3}\frac{a_2-a_1}{a_3-a_2},$$

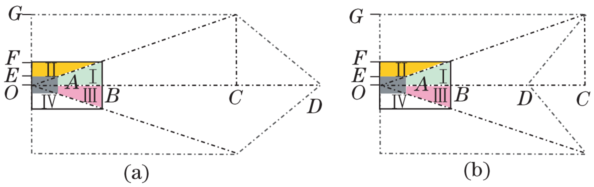

Fig. 1. Schematic demonstration of how a small object is transformed into a larger object. Transformation of a small rectangle into a large (a) convex and (b) concave pentagon.

$$c=\frac{\partial y'}{\partial x}=\frac{a_2(a_2-a_1)(a_3-a_1)}{p(a_3-a_2)\{a_2[(a_3-a_1)]-(a_3-a_2)x'\}},$$

$$d=\frac{\partial y'}{\partial y}=-\frac{a_2-a_1}{a_3-a_2}+\frac{a_2(a_2-a_1)(a_3-a_1)}{(a_3-a_2)\{a_2[(a_3-a_1)]-(a_3-a_2)x'\}}\pm\frac{a_2(a_2-a_1)(a_3-a_1)y'}{p(a_3-a_2)\{a_2[(a_3-a_1)]-(a_3-a_2)x'\}},$$

$$p=-\frac{a_2-a_1}{a_3-a_2}+\frac{a_2(a_2-a_1)(a_3-a_1)}{x'\{a_2[(a_3-a_1)]-(a_3-a_2)x'\}}.$$

The transformation from $\bar{\mu}$ $(\bar{\varepsilon})$ to $\bar{\bar{\mu}}$ $(\bar{\bar{\varepsilon}})$ is omitted. The transformation equations in Region II are

$$\begin{cases} x'=-\dfrac{b_2-b_1}{b_3-b_2}x+b_2\dfrac{b_3-b_1}{b_3-b_2}\dfrac{x}{y} \\ y'=-\dfrac{b_2-b_1}{b_3-b_2}y+b_2\dfrac{b_3-b_1}{b_3-b_2} \\ z'=z \end{cases} \tag{3}$$

By straightforward calculations, the relative permittivity and permeability in Region II are

$$\frac{\bar{\bar{\varepsilon}}}{\varepsilon}=\frac{\bar{\bar{\mu}}}{\mu}=\begin{bmatrix} \dfrac{(b_3-b_2)^2y'^4+(b_3-b_1)^2b_2^2x'^2}{y'^3m} & n & 0 \\ n & \dfrac{m}{(b_3-b_2)^2y'} & 0 \\ 0 & 0 & \dfrac{m}{(b_2-b_1)^2y'} \end{bmatrix}, \tag{4}$$

where $m=(b_3-b_2)^2y'-(b_3-b_2)(b_3-b_1)b_2$ and $n=-\frac{(b_3-b_1)b_2x'}{(b_3-b_2)y'^2}$. The permittivity $\bar{\bar{\varepsilon}}$ and permeability $\bar{\bar{\mu}}$ in Regions III and IV are similar to those in Regions I and II due to the symmetry of our device.

For convenience, the parameters are first given as follows. The frequency of the incident wave is 0.95 GHz in Figs. 2(a) and (a′), 0.65 GHz in Figs. 2(b) and (b′), 0.5 GHz in Figs. 2(c) and (c′), and 2 GHz in Fig. 3. In Fig. 2(a′), the values of the constitutive parameters are from Ref. [39]. In Figs. 2(b′) and 3(b), the values of the constitutive parameters are as follows: $|OA|=0.05$ m, $|OB|=0.1$ m, $|OC|=0.2$ m, $|CD|=0.1$ m, $|OE|=0.025$ m, $|OF|=0.05$ m, and $|OG|=0.15$ m. In Figs. 2(c′) and 3(e), the values of the constitutive parameters are as follows: $|OA|=0.05$ m, $|OB|=0.1$ m, $|OC|=0.3$ m, $|CD|=0.1$ m, $|OE|=0.025$ m, $|OF|=0.05$ m, and $|OG|=0.15$ m. The structure of Figs. 2(b′) and 3(b) corresponds to Fig. 1(a) and the structure of Figs. 2(c′) and 3(e) corresponds to Fig. 1(b).

To demonstrate our idea, a small waveguide was designed to replace a big virtual tunnel based on finite element simulations. A transverse electric wave was used as the incident wave polarized along the z axis. In Fig. 2(a), part (a rectangle) of a continuous waveguide was cut off, dividing the waveguide into two separate ports. A large amount of energy was dispersed into the space and only a small portion of the energy was transported into the end port. when a 0.03×0.06 (m) rectangular waveguide coated with transformation media was placed between the two ports, the EM wave from the left port could pass through the middle virtual space and then arrive at the right port without any loss, as shown in Fig. 2(a′). Now, we cut a convex (or concave) pentagon

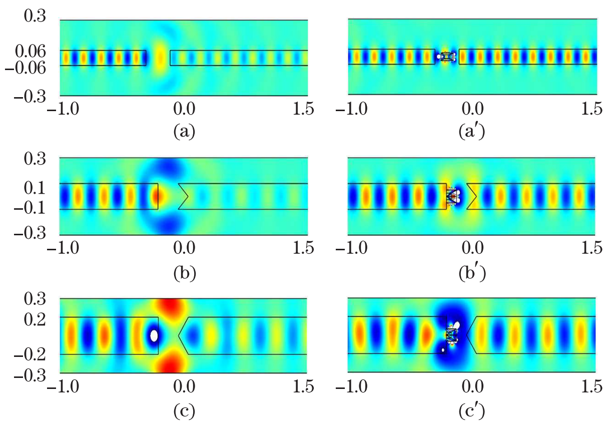

Fig. 2. The electric field distribution when the EM wave is transmitted from the left port to the right port with (a) rectangle, (b) convex pentagon, and (c) concave pentagon in the open space between these two separate ports. The electric field distribution of the EM wave in such a system when the small rectangle waveguide coated with transformation medium is placed between the two separate ports, behaves as a large invisible (a′) rectangle, (b′) convex pentagon, and (c′) concave pentagon tunnel connecting the two ports. The unit is meter.

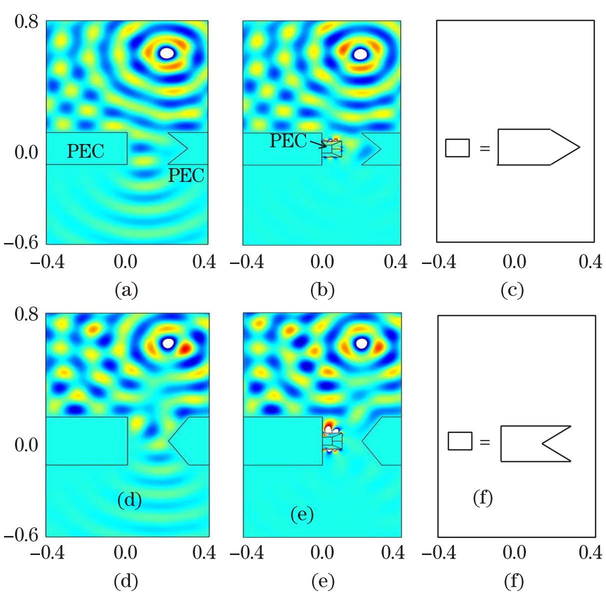

Fig. 3. The electric field distribution for an electric line source near the (a) convex and (d) concave pentagon entrance. The (b) convex and (e) concave pentagon entrance is concealed by the superscatterer. (c) and (f) Schematic diagrams representing the functions of (b) and (e) respectively.

part from the continuous waveguide (Figs. 2(b) or (c)). Most of the EM field energy from the left port was dispersed into the open space. When the small rectangle waveguide on the right side of the left port was coated with transformation media, the energy of the EM wave from the left port could tunnel from the virtual space to the right port completely (Figs. 2(b′) and (c′)). The results show that the small rectangular waveguide can be amplified into a large convex pentagon waveguide, as shown in Fig. 2(b′), which is different from the previous work in Ref. [38]. Figure 2(c′) also shows that a small rectangular waveguide can be amplified into a large concave pentagon waveguide based on the transformation optics theory.

The design of a hidden portal is demonstrated, in which a big entrance is concealed from EM wave detection (Figs. 3(c) and (f)). An electric line source is positioned behind the big convex and concave pentagon entrance, and the corresponding field distributions are shown in Figs. 3(a) and (d), respectively. The EM wave passed through the big convex (or concave) pentagon entrance. When a small rectangular perfect electric conductor (PEC) coated with transformation media was placed in the left side of the entrance, the EM wave could not pass through the virtual space due to the superscatterer, as shown in Figs. 3(b) and (e). The rectangular PEC can be amplified into a large convex (or concave) pentagon entrance, which is different from the case in Ref. [39], wherein a small rectangle was just amplified into a large rectangle. Therefore, our results are more general than those in Ref. [39].

In summary, we have shown how, with the use of complementary media theory, a small object can be designed to transform into a big object. Two typical examples, i.e., transforming a small waveguide into a large waveguide and concealing a large entrance, have been discussed based on detailed numerical simulations. The two devices may have some practical applications in manipulating EM fields.

This work was supported by the Leading Academic Discipline Project of Shanghai Normal University (No. DZL810) and the Program for Sciences and Technology Project of Shanghai Normal University (Nos. SK200857 and A-3103-10-003).

References

1. A. Alù and N. Engheta, Phys. Rev. E **72,** 016623 (2005).
2. J. B. Pendry, D. Schurig, and D. R. Smith, Science **312,** 1780 (2006).
3. U. Leonhardt, Science **312,** 1777 (2006).
4. S. A. Cummer, B. I. Popa, D. Schurig, and D. R. Smith, Phys. Rev. E **74,** 036621 (2006).
5. D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314,** 977 (2006).
6. W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, Nature Photon. **1,** 224 (2007).
7. H. Chen, B. I. Wu, B. Zhang, and J. A. Kong, Phys. Rev. Lett. **99,** 063903 (2007).
8. B. Zhang, H. Chen, B. I. Wu, and J. A. Kong, Phys. Rev. Lett. **100,** 063904 (2008).
9. Z. Ruan, M. Yan, C. W. Neff, and M. Qiu, Phys. Rev. Lett. **99,** 113903 (2007).
10. M. Yan, Z. Ruan, and M. Qiu, Phys. Rev. Lett. **99,** 233901 (2007).
11. X. Zhou and G. Hu, Phys. Rev. E **75,** 046606 (2007).
12. A. Nicolet, F. Zolla, and S. Guenneau, Opt. Lett. **33,** 1584 (2008).
13. D. H. Kwon and D. H. Werner, Appl. Phys. Lett. **92,** 013505 (2008).
14. P. Zhang, Y. Jin, and S. He, Appl. Phys. Lett. **93,** 243502 (2008).
15. H. Ma, S. Qu, Z. Xu, J. Zhang, B. Chen, and J. Wang, Phys. Rev. A **77,** 013825 (2007).
16. S. Zhang, D. A. Genvo, C. Sun, and X. Zhang, Phys. Rev. Lett. **100,** 123002 (2008).
17. P. Yao, Z. Liang, and X. Jiang, Appl. Phys. Lett. **92,** 031111 (2008).
18. W. Jiang, T. Cui, Q. Cheng, J. Y. Chin, X. M. Yang, R. Liu, and D. R. Smith, Appl. Phys. Lett. **92,** 264101 (2008).
19. C. Li and F. Li, Opt. Express **16,** 13414 (2008).
20. T. Tang, F. Chen, and B. Sun, Chin. Opt. Lett. **8,** 431 (2010).
21. S. Ma, W. B. Cui, D. Li, N. K. Sun, D. Y. Geng, X. Jiang, and Z. D. Zhang, Appl. Phys. Lett. **92,** 173113 (2008).
22. J. Hu, X. Zhou, and G. Hu, Appl. Phys. Lett. **95,** 011107 (2009).
23. H. Ren, Y. Qin, K. Liu, Z. Wu, W. Hu, C. Jiang, and Y. Jin, Chin. Opt. Lett. **8,** 749 (2010).
24. X. Xu, Y. Feng, Y. Hao, J. Zhao, and T. Jiang, Appl. Phys. Lett. **95,** 184102 (2009).
25. C. Qiu, L. Hu, B. Zhang, B. I. Wu, S. G. Johnson, and J. D. Joannopoulos, Opt. Express **17,** 13467 (2009).
26. X. Zang and C. Jiang, Opt. Express **18,** 10168 (2010).
27. X. Zang and C. Jiang, Opt. Express **18,** 6891 (2010).
28. H. Chen, B. Hou, S. Chen, X. Ao, W. Wen, and C. T. Chan, Phys. Rev. Lett. **102,** 183903 (2009).
29. Y. Luo, J. Zhang, H. Chen, J. Huangfu, and L. Ran, Appl. Phys. Lett. **95,** 193506 (2009).
30. H. Chen and C. T. Chan, Appl. Phys. Lett. **90,** 241105 (2007).
31. M. Schmiele, C. Rockstuhl, and F. Lederer, Phys. Rev. A **79,** 053854 (2009).
32. M. Rahm, S. A. Cummer, D. Schurig, J. B. Pendry, and D. R. Smith, Phys. Rev. Lett. **100,** 063903 (2008).
33. H. Chen and C. T. Chan, Appl. Phys. Lett. **91,** 183518 (2007).
34. T. Yang, H. Chen, X. Luo, and H. Ma, Opt. Express **16,** 18545 (2008).
35. J. Ng, H. Chen, and C. T. Chan, Opt. Lett. **34,** 644 (2009).
36. Y. Lai, H. Chen, Z. Zhang, and C. T. Chan, Phys. Rev. Lett. **102,** 093901 (2009).
37. Y. Lai, J. Ng, H. Chen, D. Han, J. Xiao, Z. Zhang, and C. T. Chan, Phys. Rev. Lett. **102,** 253902 (2009).
38. J. Zhang, Y. Luo, H. Chen, J. Huangfu, B. I. Wu, L. Ran, and J. A. Kong, Opt. Express. **17,** 6203 (2009).
39. X. Luo, T. Yang, Y. Gu, H. Chen, and H. Ma, Appl. Phys. Lett. **94,** 223513 (2009).