Multi-cavity-stabilized ultrastable laser

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We demonstrate a proposal for making an ultrastable laser referenced to a multi-cavity, enabling a lower thermal noise limit due to the averaging effect. In comparison with a single-cavity system, relative frequency instability of the synthesized laser can be improved by a factor of the square root of the cavity number. We perform an experiment to simulate a two-cavity system with two independent ultrastable lasers. Experimental results show that the relative frequency instability (Allan deviation) of the synthesized laser is 5×10^{-16}, improved by a factor of $\sqrt{2}$ from a single-cavity-stabilized laser.

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An ultrastable laser (U) is a vital component in many scientific applications, including optical atomic clocks[1], gravitational wave detection with laser interferometry[2], dark matter detection[3,4], and verification of physics laws[5]. State-of-the-art Us exhibit frequency instability at the 10^{-17} level, limited by the thermal noise effect[6–9]. However, the quantum projection noise level of the best optical clocks has reached 10^{-18} at the 1 s level[10,11]. With the best quantum noise limited optical atomic clocks, the newly proposed gravitational wave detection proposal becomes more realistic[12]. More stable laser systems are required.

The main contribution of the thermal effect comes from the mirror displacement (ΔL) due to Brownian motion. According to the fluctuation dissipation theorem (FDT)[13], a mirror's thermal noise effect described as the double-sideband power spectrum is given in Ref. [14]:

$$G_{\text{mirror}}(f) = \frac{4k_B T}{\omega}\frac{1-\sigma^2}{\sqrt{\pi}Ew_0}\left(\phi_{\text{sub}} + \frac{2}{\sqrt{\pi}}\frac{1-2\sigma}{1-\sigma}\frac{d}{w_0}\phi_{\text{coat}}\right), \tag{1}$$

where k_B is the Boltzmann constant, T is the temperature, and $\omega = 2\pi f$ is the angular frequency; σ, E, and ϕ_{sub} are Poisson's ratio, Young's modulus, and mechanical loss of the mirror substrate, respectively; d and ϕ_{coat} are the thickness and mechanical loss of the coating; w_0 is the beam radius of a Gaussian beam. The frequency instability of a cavity-stabilized laser is determined by the length instability of the reference cavity; the mirror's thermal noise effect on laser can be written as

$$\begin{aligned}\frac{\sqrt{S_\nu(f)}}{\nu} &= \frac{\sqrt{2G_{\text{mirror}}(f)}}{L}\\ &= \sqrt{\frac{4k_B T}{\pi^{3/2} f}\frac{1-\sigma^2}{Ew_0L^2}\left(\phi_{\text{sub}} + \frac{2}{\sqrt{\pi}}\frac{1-2\sigma}{1-\sigma}\frac{d}{w_0}\phi_{\text{coat}}\right)},\end{aligned} \tag{2}$$

where $S_\nu(f)$ is the frequency noise, ν is the laser frequency, and the factor of 2 in front of $G_{\text{mirror}}(f)$ represents two mirrors of a cavity. This flicker noise corresponds to the relative frequency instability independent of averaging time, indicated by the Allan deviation[15,16]:

$$\begin{aligned}\sigma_A &= \sqrt{2\ln 2 f}\frac{\sqrt{S_\nu(f)}}{\nu}\\ &= \sqrt{\ln 2\frac{8k_B T}{\pi^{3/2}}\frac{1-\sigma^2}{Ew_0L^2}\left(\phi_{\text{sub}} + \frac{2}{\sqrt{\pi}}\frac{1-2\sigma}{1-\sigma}\frac{d}{w_0}\phi_{\text{coat}}\right)}.\end{aligned} \tag{3}$$

Equation (3) contains all factors that determinate the thermal noise effect. Over the last decade, efforts have been made to reduce this effect by using a long cavity[8], applying low mechanical loss materials[17–20], cooling the cavity[7,19], or enlarging the beam size[11,21]. It is difficult to have a cavity longer than a half meter, because of the lack of such a bulky ultralow expansion (ULE) glass and the complexity for reducing the vibration sensitivity and temperature fluctuation. Up to now, the longest ultrastable cavity has been 48 cm[8]. Regarding the choice of the mirror substrate, ULE glass, fused silica, single-crystal silicon, and special coatings have been investigated. Fused silica exhibits a smaller mechanical loss than ULE glass[17,18]; single-crystal silicon has a larger Young's modulus than fused silica and ULE, and cavities entirely made of it have shown great thermodynamic performance[19]; monocrystalline multilayer coatings have smaller mechanical loss than dielectric multilayer coatings[20]. Providing a cryogenic environment for the cavity is a straightforward way of reducing the thermal noise. Recently, a 21-cm-long silicon cavity at 124 K and a 6-cm-long silicon cavity at 4 K exhibit a relative instability of 4×10^{-17} and 1×10^{-16}[6,7]. Enlarging the laser beam size can be achieved by using a concave mirror with a larger radius of curvature[11,21]. However, a larger radius cavity leads to difficulties of mode matching due to high-order modes being excited more easily[22].

It seems that scientists have considered all possible factors shown in Eq. (3), including cavity length, temperature, cavity materials, and beam size, to reduce the thermal noise effect of a two-mirror cavity. An alternative way is use of a fold cavity, because its equivalent cavity length is longer[23]. For example, a four-mirror fold cavity [see Fig. 1(a)] can reduce the thermal effect by a factor of approximately $\sqrt{5/3} \approx 0.75$, assuming that all mirrors have the same thermal noise level and that the tilt angle of light is small. However, this idea may not be possible to realize due to difficulties in making and mounting the tilt mirrors. Considering that the photons bound back and forth between both sides of the spacer, we also consider that the fold cavity has a larger equivalent beam size. The question arises of why we do not mount several [$n = 2$ in this case; see Fig. 1(b)] separated cavities onto a common spacer. If one can average the lengths of these cavities, the equivalent beam size is then enlarged n times; consequently, the relative length instability gains $\sqrt{n}$ ($1/\sqrt{2} \approx 0.71$ in this case), because the thermal noise on different cavities is not coherent. One can expect that a 21-cm-long silicon cavity operating at 4 K rather than at 124 K should exhibit a relative length instability 5–6 times lower than 4×10^{-17}; considering that some technical noise may appear at this level, the laser instability should be close to 1×10^{-17}. If one can make a similar multi-cavity system to further improve the laser instability, 10^{-18} is also possible.

Now the question becomes how to average the length of cavities, i.e., frequency of lasers, performing a laser with a frequency of

$$\nu_{1,n} = \frac{1}{n}\sum_{i=1}^{n} \nu_i, \tag{4}$$

where n is the number of cavities, ν_i is the laser frequency. Direct optical synthesizing by the nonlinear effect is difficult, especially for the dividing process. Fortunately, we can subtract between lasers by using a photodetector, and sum the laser and radio frequency (RF) by using an acoustic-optical modulator (AOM). Thus, we can perform such an optical frequency synthesizing by employing mature RF techniques. Figure 2 shows the schematic of the laser synthesizer. Lasers should be frequency-stabilized onto cavities first; then, we can combine one laser (laser 1) with all other lasers and direct them to photodetectors, in which heterodyne signals ($\nu_2 - \nu_1$, $\nu_3 - \nu_1, \ldots$) can be produced. Frequency summation by RF mixers yields the sum frequency of $\nu_2 + \cdots + \nu_n - (n-1)\nu_1$. Then, one can frequency divide this signal to obtain $[\nu_2 + \cdots + \nu_n - (n-1)\nu_1]/n$. To match the resonant frequency of the AOM, an additional frequency shift (ν_{SG}) can be provided by a low-noise signal generator (SG). Finally, the synthesized laser $\frac{1}{n}\sum_{i=1}^{n} \nu_i - \nu_{\mathrm{SG}}$ is produced at the output of the AOM. Note that the bandwidth of the photodetectors is usually higher than 2 GHz and can be beyond 10 GHz with fast ones.

To verify this technique, we perform an experiment to simulate a two-cavity system by using two identical but independent cavity-stabilized lasers. Figure 3 shows the experimental setup. The Us, based on commercial 1555 nm lasers, exhibit a frequency instability of 7×10^{-16} and a linewidth of ~100 mHz, mainly limited by the thermal noise effect of the 10 cm ULE cavity[24]. The beatnote between two lasers ($\nu_2 - \nu_1 = 1.70$ GHz) is detected by a photodiode (PD, ET3000A, Electro-Optics Technology) and sent into a frequency divider with a factor of 2 (based on MC10EP139, ON Semiconductor), and then, we obtain the divided frequency $(\nu_2 - \nu_1)/2$ at 0.85 GHz. Next, we down-convert this signal by mixing

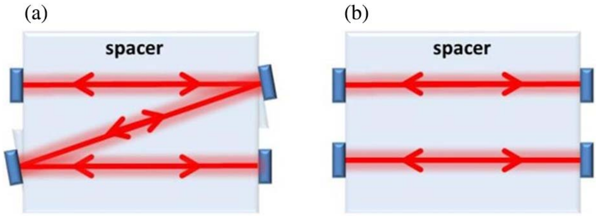

Fig. 1. Diagrammatic sketch of (a) fold cavity and (b) muti-cavity.

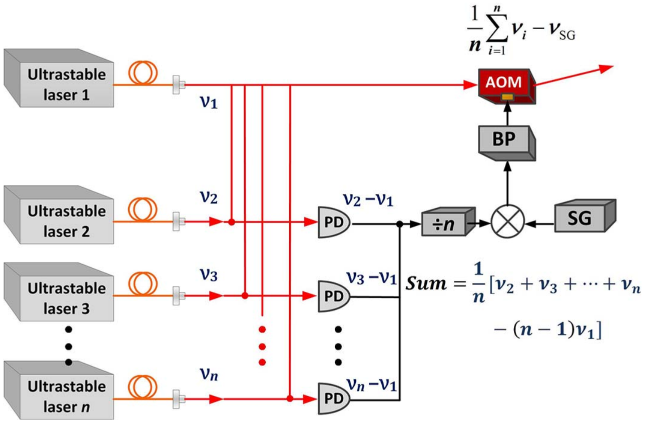

Fig. 2. Schematic of the optical synthesizer. PD, photodiode; SG, signal generator; ÷n, frequency divider; BP, band-pass filter; AOM, acoustic-optical modulator.

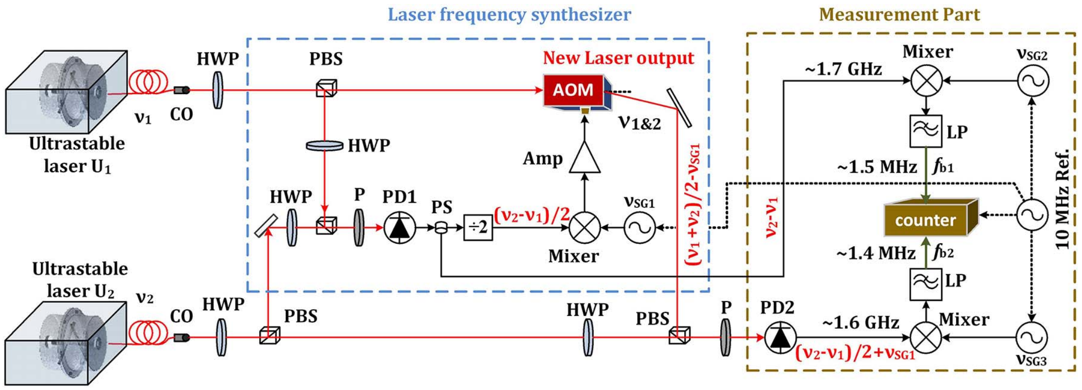

Fig. 3. Experimental setup of an ultrastable laser with two reference cavities. U, ultrastable laser; CO, collimator; PBS, polarization beam splitter; HWP, half-wave plate; P, polarizer; PD, photodiode; ÷2, frequency divider; SG, signal generator; LP, low pass filter; PS, RF signal power splitter; red line, optical signal; black line, electronic signal.

with a reference frequency ν_{SG1} (0.74 GHz) provided by a SG (SG382, Stanford Research Systems) to 110 MHz, which is the resonant frequency of the tank circuits of the AOM (MGAS110-A1, AA Opto Electronic). At the same time, the other part of laser from U1 passes through the AOM via Bragg diffraction[25]. By picking up the positive first-order diffraction light, we can obtain the synthesized laser with a frequency of

$$\nu_{1\&2} = \nu_2 + \nu_{\mathrm{AOM}} = (\nu_1 + \nu_2)/2 - \nu_{\mathrm{SG1}}. \tag{5}$$

To measure the additional noise introduced by the optical frequency synthesizing process, we produce another beatnote between ν_1 and $\nu_{1\&2}$, i.e., $(\nu_2 - \nu_1)/2 +$ 0.85 GHz, as shown in Fig. 3. We down-convert two optical beatnotes to the operational range of a multi-channel frequency counter (FXQE80, $K + K$ Messtechnik) by mixing them with reference frequencies given by SGs. We expect that the variation of these two down-converted signals $s(f_{b1}, f_{b2})$ has a ratio of 2 if the frequency synthesizing does not introduce considerable noise effect. The discrimination $(f_{b1} - 2f_{b2})$ is attributed to the frequency instability due to the frequency synthesizing, which indicates the limit of such a technique. Note that we synchronize SG1, SG2, SG3, and the frequency counter to a common 10 MHz reference frequency.

As shown in Fig. 4, the relative frequency instability of two independent lasers is about 1×10^{-15} for 1–20 s integration time, while that of one laser (ν_1) against the synthesized laser $(\nu_1 + \nu_2)/2 - \nu_{\mathrm{SG}}$ is at the 5×10^{-16} level, as expected. This also indicates frequency instability of the synthesized laser, because ν_1 and ν_2 are identical and independent, so that the deviations of $(\nu_2 - \nu_1)/2$ and $(\nu_1 + \nu_2)/2$ are the same. The frequency instability is 7×10^{-17} at 1 s and rolls down to 1×10^{-17} at 16 s, which is attributed to the two channels of the frequency counter that are not synchronized perfectly. In order to verify this, we replace the beatnote signal from PD1 with a low-noise 1.7 GHz RF signal, and the beatnote signal from PD2 with the signal directed to the AOM. Then, we obtain three noise floor levels corresponding to the three upper curves. All of the noise floors are in the range of 10^{-18} and 10^{-19}. Since frequency instability no longer agrees with the ratio of 2, we conclude that the noise floor is mainly determined by the SGs. A lower noise effect can be realized with better RF SGs. In a crucial case, this effect could be minimized by using the photonic microwave generation technique, yielding ultralow-noise microwave signals in the 10^{-16} level, which is a few orders of magnitude better than the RF signals in this case[26].

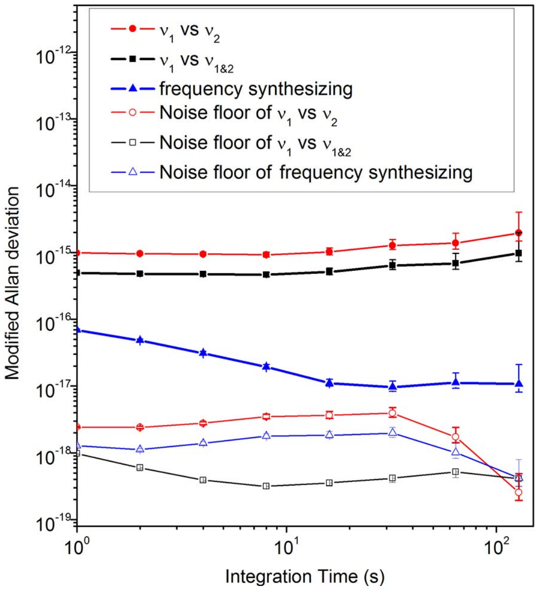

Fig. 4. Relative frequency instability of lasers and their noise floors, where ν_1 and ν_2 are frequencies of two independent ultrastable lasers, $\nu_{1\&2}$ is the synthesized laser with a frequency of $(\nu_1 + \nu_2)/2 - \nu_{\mathrm{SG1}}$, discrimination is obtained from variation of $(\nu_1 - \nu_2) - 2(\nu_1 - \nu_{1\&2})$, and normalized by the optical frequency (192 THz).

As mentioned above, a space-saving solution is to place high-finesse cavities in a common spacer, e.g., a cubic cavity body with cavities orthogonal to each other in the mid horizontal plane. Such a force-insensitive cubic cavity body with only one cavity was designed several years ago[27]. With a long cavity, it is also possible to have a stable multi-cavity with low vibration sensitivity by holding the cavity at a balanced position. In fact, it is not necessary to optimize vibration sensitivity of each cavity, but rather to optimize the sum of these cavity lengths. It is worth noting that other techniques, such as the use of spectral hole burning[28,29], can also benefit from this approach to improve a laser's frequency instability by averaging the frequencies of all lasers.

In summary, we have demonstrated a proposed approach for improving the frequency instability of Us by using a multi-cavity design. In addition, we have performed an experiment to simulate a two-cavity system. Our result shows that the frequency instability of the synthesized laser is improved by a factor of $\sqrt{2}$, and the additional frequency instability induced by frequency synthesizing is at a low level of 10^{-18}. This noise effect is attributed to the noise of RF SGs. With better RF SGs or by using a photonic microwave generation technique, this effect can be further reduced. Based on a multi-cavity (2–4 cavities) design, one can further improve frequency instability of current Us by a factor of 1.4–2, up to 10^{-18} or even better.

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